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On exotic hybrid meson production in $\gamma^*\gamma$ collisions

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Abstract. We present a theoretical study of exotic hybrid meson ($J^{PC} = 1^{-+}$) production in photon-photon collisions where one of the photons is deeply virtual, including twist two and twist three contributions. We calculate the cross section of this process, which turns out to be large enough to imply sizeable counting rates in the present high luminosity electron–positron colliders. We emphasize the importance of the $\pi\eta$ decay channel for the detection of the hybrid meson candidate $\pi_1(1400)$ and calculate the cross section and the angular distribution for $\pi\,\eta$ pair production in the unpolarized case. This angular distribution is a useful tool for disentangling the hybrid meson signal from the background. Finally, we calculate the single spin asymmetry associated with one initial longitudinally polarized lepton.

1 Introduction

Photon–photon collisions, with one deeply virtual photon, is an excellent tool for the study of different aspects of QCD. The main feature of such processes is that a QCD factorization theorem holds, which separates a hard partonic subprocess involving scattered photons from a distribution amplitude [1] describing a transition of a quark–antiquark pair to a meson or a generalized distribution amplitude [2] describing the transition of a quark–antiquark pair to two-meson or three-meson states [3]. The success of this description [4] with respect to recent LEP data [5] is indeed striking enough to propose such processes for use as a tool for the discovery of new hadronic states [6]. In this paper, we focus on the process where an exotic $J^{PC} = 1^{-+}$ isotriplet hybrid meson H (which may be the $\pi_1(1400)$ state [7]) is created in photon–photon collisions and then decays into $\pi^0\eta$. This exotic particle has been the subject of intensive studies for many years [8].

In previous papers, we have shown [9] that, contrary to naive expectations, the longitudinally polarized hybrid meson possesses a leading twist distribution amplitude (DA) related to the usual non-local quark-antiquark correlators. We have also been able to estimate the normalization of this DA, which is by no means small, with respect to the one for usual non-exotic mesons. We thus advocate that exclusive deep electroproduction processes will be able to copiously produce these exotic states. In this paper, we extend our analysis of [9] to $\gamma^*\gamma$ collisions with the production of both longitudinally and transversally polarized hybrid mesons. We calculate the hard amplitude up to the level of twist three and thus ignore the contributions of mass terms. Indeed, the case of hybrid production and its decay products, the $\pi\eta$ pair in electron-photon collisions, is similar to the electron-proton case, with the important distinction that no unknown generalized parton distribution enters the amplitude, so that the only place where non-perturbative physics enters is the final state DA or generalized distribution amplitudes (GDA). We emphasize the $\pi\eta$ pair production as a promising way for detecting the hybrid meson.

Since we are dealing with the production of an isovector state, there is no mixing between quark–antiquark correlators and gluon–gluon correlators. Since lepton beams are easily polarized, we also consider the single spin asymmetry associated with the case where one of the initial leptons is longitudinally polarized, while the polarizations of the other one are averaged over. This single spin asymmetry will turn out to give access to the phase difference of leading twist and twist three components of the final state GDA.

Finally let us note that in the case of the electron-photon collisions producing a $\pi\eta$ pair, i.e. a state with the positive charge parity $C = +$, the bremsstrahlung contribution does not exist.

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2 Vacuum-to-hybrid meson, vacuum-to-hadron matrix elements and their properties

2.1 The hybrid meson distribution amplitude

Let us first consider relevant vacuum-to-hybrid meson matrix elements that enter in exclusive hard amplitudes. We suppose that the hybrid meson is in the isotriplet state with $J^{PC} = 1^{-+}$ quantum numbers. A meson with such quantum numbers should be built of quarks and gluons, that is one should work beyond the quark-antiquark model. We have shown [9] that in the case of the longitudinal hybrid meson polarization, the leading contribution to a hard amplitude comes from the non-local quark-antiquark correlators. The non-local character of such correlators leads to the gluonic degrees of freedom included in the gauge-invariant link.

Let us form the light-cone basis adapted for $\gamma^*\gamma \to H$ and $\gamma^*\gamma \to \pi\eta$ processes. We introduce the "large" and "small" vectors as

$$\begin{aligned} n^* &= (\Lambda, \mathbf{0}_T, \Lambda)\,, \\ n &= \left(\frac{1}{2\Lambda}, \mathbf{0}_T, -\frac{1}{2\Lambda}\right), \\ n^*\cdot n &= 1\,, \end{aligned} \tag{1}$$

respectively, and express the photons and hybrid meson momenta via the Sudakov decomposition:

$$\begin{aligned} q &= n^* - \frac{Q^2}{2}n\,, \\ q' &= \frac{M_H^2+Q^2}{2}n\,, \\ p &= n^* + \frac{M_H^2}{2}n\,. \end{aligned} \tag{2}$$

We define the transverse tensor $g^T_{\mu\nu} = g_{\mu\nu} - n^*_\mu n_\nu - n_\mu n^*_\nu$. Using the results of seminal studies on the DAs of vector mesons [10], we write ($\bar{u} = 1-u$):

$$\begin{aligned} &\langle H(p,\lambda)|\bar{\psi}(z)\gamma_\mu[z;-z]\psi(-z)|0\rangle = \\ &f_H M_H \Bigg[p_\mu \mathrm{e}^{(\lambda)}\cdot n \int_0^1 \mathrm{d}u\, \mathrm{e}^{i(u-\bar{u})p\cdot z}\phi_1^H(u) \\ &\quad + \mathrm{e}^{(\lambda)}_{\mu T}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}(u-\bar{u})p\cdot z}\phi_3^H(u)\Bigg] \end{aligned} \tag{3}$$

for the vector correlator, and

$$\begin{aligned} &\langle H(p,\lambda)|\bar{\psi}(z)\gamma_\mu\gamma_5[z;-z]\psi(-z)|0\rangle = \\ &\mathrm{i} f_H M_H \varepsilon_{\mu e_T^{(\lambda)} pn}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}(u-\bar{u})p\cdot z}\phi_A^H(u) \end{aligned} \tag{4}$$

for the axial correlator. We use the following short notation: $\varepsilon_{s\,k\,m\,l} = \varepsilon_{\mu_1\mu_2\mu_3\mu_4}s_{\mu_1}k_{\mu_2}m_{\mu_3}l_{\mu_4}$. In (3) and (4), the polarization vector $\mathrm{e}^{(\lambda)}_\mu$ describes the spin state of the hybrid meson. Due to C-charge invariance, the symmetry properties of DAs are manifested in the following relations:

$$\begin{aligned} \phi_1^H(u) &= -\phi_1^H(1-u)\,, \\ \phi_3^H(u) &= -\phi_3^H(1-u)\,, \\ \phi_A^H(u) &= \phi_A^H(1-u)\,. \end{aligned} \tag{5}$$

Compared to the ρ meson matrix elements [10], one can see that the corresponding DAs for the exotic hybrid meson has different symmetry properties.

The leading twist longitudinally polarized hybrid meson DA has been discussed in [9] and shown to be asymptotically equal to

$$\phi_1^H(u) = 30u(1-u)(1-2u)\,, \tag{6}$$

and f_H to be of the order of 50 MeV. In the present work, we will not include any QCD evolution effects in GDA.

In order to be able to estimate twist three contributions, we have to build a model for the twist three DAs ϕ_3^H and ϕ_A^H. We will not innovate on this point but restrict ourselves to the Wandzura–Wilczek parts [11], which read [10]:

$$\begin{aligned} \phi_3^{WW}(u) &= \frac{1}{2}\left[\int_0^u \mathrm{d}v\frac{\phi_1(v)}{v-1} - \int_u^1 \mathrm{d}v\frac{\phi_1(v)}{v}\right], \\ \phi_A^{WW}(u) &= \frac{1}{2}\left[\int_0^u \mathrm{d}v\frac{\phi_1(v)}{v-1} + \int_u^1 \mathrm{d}v\frac{\phi_1(v)}{v}\right], \end{aligned} \tag{7}$$

yielding for our choice of ϕ_1^H:

$$\begin{aligned} \phi_3^{WW}(u) &= -\frac{5}{2}(1-2u)^3\,, \\ \phi_A^{WW}(u) &= \frac{5}{2}[1+6u(u-1)]\,. \end{aligned} \tag{8}$$

2.2 The $\pi\eta$ generalized distribution amplitude

We now come to a discussion of the $\pi\eta$ GDA that enters the amplitude if we try to detect the hybrid meson through its $\pi\eta$ decay mode, which may be the dominant one for the $\pi_1(1400)$ state. Using the results of [12], we have

$$\begin{aligned} &\langle\pi^0(p_\pi)\eta(p_\eta)|\bar{\psi}(-z)\gamma^\mu[-z;z]\psi(z)|0\rangle = \\ &P^\mu_{\pi\eta}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}(\bar{u}-u)P_{\pi\eta}\cdot z}\Phi_1^{(\pi\eta)}\left(u,\zeta,m^2_{\pi\eta}\right) \\ &\quad + \Delta^{\mu T}_{\pi\eta}\int_0^1 \mathrm{d}u\, \mathrm{e}^{\mathrm{i}(\bar{u}-u)P_{\pi\eta}\cdot z}\Phi_3^{(\pi\eta)}\left(u,\zeta,m^2_{\pi\eta}\right) \end{aligned} \tag{9}$$

for the vector correlator, and

$$\begin{aligned} &\langle\pi^0(p_\pi)\eta(p_\eta)|\bar{\psi}(-z)\gamma^\mu\gamma_5[-z;z]\psi(z)|0\rangle = \\ &\varepsilon^{\mu\alpha\beta n}\Delta^{\alpha T}_{\pi\eta}P^\beta_{\pi\eta}\int_0^1 \mathrm{d}u\, \mathrm{e}^{i(\bar{u}-u)P_{\pi\eta}\cdot z}\Phi_A^{(\pi\eta)}\left(u,\zeta,m^2_{\pi\eta}\right) \end{aligned} \tag{10}$$

for the axial correlator. In (9) and (10) the total momentum and relative momentum of $\pi\eta$ pair are

$$P_{\pi\eta} = p_\pi + p_\eta = n^* + \frac{m^2_{\pi\eta}}{2} n,$$
$$\Delta_{\pi\eta} = p_\pi - p_\eta = \left(2\zeta - 1 + \frac{m_\pi^2 - m_\eta^2}{m^2_{\pi\eta}}\right) P_{\pi\eta} + (1-2\zeta) m^2_{\pi\eta} n + \Delta^T_{\pi\eta}\,. \quad (11)$$

The $\pi\eta$ leading twist GDA has been discussed in [9] to be in the $J^{PC} = 1^{-+}$ channel

$$\tilde{\Phi}_1^{(\pi\eta)}\left(u,\zeta,m^2_{\pi\eta}\right) = 30u(1-u)(1-2u) B_{11}\left(m^2_{\pi\eta}\right) P_1(\cos\theta)\,, \quad (12)$$

with $\cos\theta = (2\zeta-1)/\beta$, $\beta = \lambda(m^2_{\pi\eta}, m^2_\pi, m^2_\eta)/m^2_{\pi\eta}$, and the coefficient function $B_{11}(m^2_{\pi\eta})$ related to the Breit–Wigner amplitude when $m^2_{\pi\eta}$ is in the vicinity of M_H^2 as

$$B_{11}\left(m^2_{\pi\eta}\right)\Big|_{m^2_{\pi\eta}\approx M_H^2} = \frac{5}{3}\frac{g_{H\pi\eta} f_H M_H \beta}{M_H^2 - m^2_{\pi\eta} - i\Gamma_H M_H}\,. \quad (13)$$

The formalism of GDA includes a background in a natural way, i.e. the contribution of final states uncorrelated with the main signal of hybrid meson exchange. We will model this background as a $J=0$ contribution (i.e. ζ-independent) without structure in $m^2_{\pi\eta}$, and with the asymptotic u-dependence (i.e. $u(1-u)(2u-1)$). We know nothing of its phase other than that it should be quite constant in the considered region. For simplicity, and contrary to the analysis of [9], we do not include any $J=2$ component in the GDA. Thus our model for the leading twist $\pi\eta$ GDA reads

$$\Phi_1^{(\pi\eta)}\left(u,\zeta,m^2_{\pi\eta}\right) = 30u(1-u)(1-2u)\left(K e^{\mathrm{i}\alpha} + B_{11}\left(m^2_{\pi\eta}\right)\cos\theta\right)\,. \quad (14)$$

Note that the GDAs $\Phi_3^{(\pi\eta)}(u,\zeta,m^2_{\pi\eta})$ and $\Phi_A^{(\pi\eta)}(u,\zeta,m^2_{\pi\eta})$ are twist three functions; they have a part that comes from the Wandzura–Wilczek (WW or kinematical) twist three and another part that is usually called genuinely twist three (see, for instance [12]). The WW part reads

$$\Phi_3^{WW}(u,\zeta) = -\frac{1}{4}\int_0^u \mathrm{d}y \frac{\partial_\zeta \Phi_1(y,\zeta)}{y-1} + \frac{1}{4}\int_u^1 \mathrm{d}y \frac{\partial_\zeta \Phi_1(y,\zeta)}{y}\,, \quad (15)$$

$$\Phi_A^{WW}(u,\zeta) = -\frac{1}{4}\int_0^u \mathrm{d}y \frac{\partial_\zeta \Phi_1(y,\zeta)}{y-1} - \frac{1}{4}\int_u^1 \mathrm{d}y \frac{\partial_\zeta \Phi_1(y,\zeta)}{y}\,, \quad (16)$$

which yields with our choice of the leading twist terms:

$$\Phi_3^{WW}(u,\zeta) = 5(1-2u)^3 \frac{B_{11}\left(m^2_{\pi\eta}\right)}{2\beta}$$
$$\Phi_A^{WW}(u,\zeta) = -5((1-2u)^2 - 2u(1-u))\frac{B_{11}\left(m^2_{\pi\eta}\right)}{2\beta}\,. \quad (17)$$

3 Amplitudes of $\gamma^*\gamma \to H$ and $\gamma^*\gamma \to \pi\eta$ processes

The gauge invariant expression for the $\gamma(q')\gamma^*(q) \to H(p)$ amplitude reads

$$T^{\gamma\gamma^*\to H}_{\mu\nu} = \frac{e^2\left(Q_u^2 - Q_d^2\right)}{\sqrt{2}} \times \left[\frac{1}{2} g^T_{\mu\nu} e^{(\lambda)}\cdot n \mathbf{A}_1 + \frac{e^{(\lambda)}_{\nu\,T}(p+q')_\mu}{Q^2}\mathbf{A}_2\right]\,, \quad (18)$$

with $Q_u = 2/3$ and $Q_d = -1/3$, and where the following short notations have been introduced:

$$\mathbf{A}_1 = \int_0^1 \mathrm{d}u E_-(u)\Phi_1(u)\,,$$
$$\mathbf{A}_2 = \int_0^1 \mathrm{d}u \Big(E_-(u)\Phi_3(u) - E_+(u)\Phi_A(u)\Big)\,,$$
$$\Phi_{1,\,3,\,A}(u) = f_H M_H \phi^H_{1,3,A}(u)\,,$$
$$E_\pm(u) = \frac{1}{1-u} \pm \frac{1}{u}\,. \quad (19)$$

Such an amplitude allows calculating the production cross section and the helicity density matrix of the hybrid meson. The knowledge of this helicity matrix leads to a definite angular distribution for any particular decay channel. In particular, for the process leading to the two meson $\pi\eta$ final state, the straightforward generalization of the amplitude derived in [12] reads

$$T^{\gamma\gamma^*\to\pi\eta}_{\mu\nu} = \frac{e^2(Q_u^2 - Q_d^2)}{\sqrt{2}} \times \left[\frac{1}{2} g^T_{\mu\nu}\mathbf{A}_1^{(\pi\eta)} + \frac{\left(\Delta^T_{\pi\eta}\right)_\nu (P_{\pi\eta}+q')_\mu}{Q^2}\mathbf{A}_2^{(\pi\eta)}\right]\,, \quad (20)$$

where, following to the notations (19), we introduce:

$$\mathbf{A}_1^{(\pi\eta)} = \int_0^1 \mathrm{d}u E_-(u)\Phi_1^{(\pi\eta)}(u) = -10\left[K\,e^{\mathrm{i}\alpha} + B_{11}\left(m^2_{\pi\eta}\right)\cos\theta\right]\,,$$
$$\mathbf{A}_2^{(\pi\eta)} = \int_0^1 \mathrm{d}u \Big(E_-(u)\Phi_3^{(\pi\eta)}(u) - E_+(u)\Phi_A^{(\pi\eta)}(u)\Big) = -\frac{5}{3\beta} B_{11}\left(m^2_{\pi\eta}\right)\,. \quad (21)$$

Note that the amplitudes (18) and (20) satisfy the gauge invariance condition: $q_\mu T_{\mu\nu} = q'_\nu T_{\mu\nu} = 0$ provided that we neglect terms proportional to the square of the meson mass, which is quite natural in the light-cone formalism in the twist three approximation.

4 Cross sections

The kinematics of the $\gamma^*\gamma \to \pi\eta$ process is illustrated in Fig. 1. We can now calculate the cross sections for the process $\gamma^*\gamma \to$ hybrid meson. As for the usual treatment of a pseudoscalar meson [13], it may be expressed in terms of the transition form factor $F_{H\gamma}$, which scales like $1/Q^2$ up to logarithmic corrections due to the QCD evolution of the DA (which we consistently ignore in this work). For an easy comparison with well-measured cases, we define R as the ratio of squared amplitudes for unpolarized photons,

$$R = \frac{T_{\mu\nu}(\gamma\gamma^* \to H)T^{*\mu\nu}(\gamma\gamma^* \to H)}{T_{\mu\nu}(\gamma\gamma^* \to \pi^0)T^{*\mu\nu}(\gamma\gamma^* \to \pi^0)}\,. \quad (22)$$

As shown in Fig. 2 by the solid line, the twist two transition form factor is sizeable in the hybrid case, of the order of the corresponding quantity for π^0 or η meson production. We are thus confident that a good detector such as those existing in the present e^+e^- colliders will be able to detect the hybrid signal if one of the decay channels has a fairly large branching ratio. To estimate the twist three effects, we now approximate the twist three part of the hybrid distribution amplitude in the Wandzura–Wilczek way [11] as explained in Sect. 2. The resulting ratio R with the twist two and twist three contributions to the hybrid transition form factor (but only the twist two contributions to the π^0 case) is shown in Fig. 2 by the dashed line. The twist three contribution is negative and of the order of 20% when $Q \approx 1$ GeV, but quite negligible when $Q \geq 3$ GeV. Let us stress that this estimate is in the WW approximation, which is by no means a proven result. Calculating the order of magnitude of the genuine twist three hybrid DA is very model-dependent and, to our knowledge, no estimate exists in the literature.

The cross section for the e^+e^- process

$$e(k_1) + e(l_1) \to e(k_2) + e(l_2) + H(p) \quad (23)$$

for unpolarized lepton beams is easily obtained when the leptonic parts are included. Note that the positive C parity of the hybrid meson does not allow any contribution from a bremsstrahlung process. Specifying a positive C parity two body decay channel, like $\pi^0\eta$, we obtain the differential cross section for the complete process (see Fig. 1)

$$e(k_1) + e(l_1) \to e(k_2) + e(l_2) + \pi(p_\pi) + \eta(p_\eta)\,, \quad (24)$$

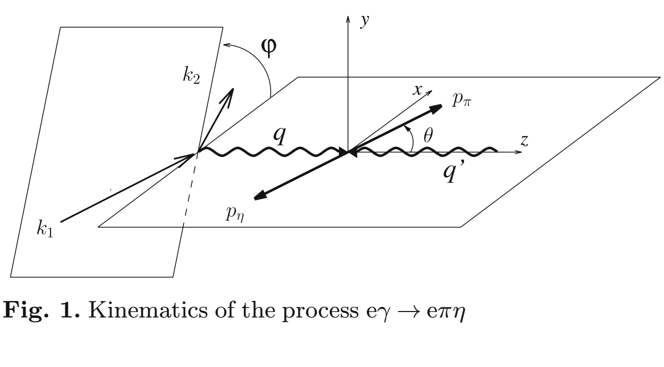

Fig. 1. Kinematics of the process $e\gamma \to e\pi\eta$

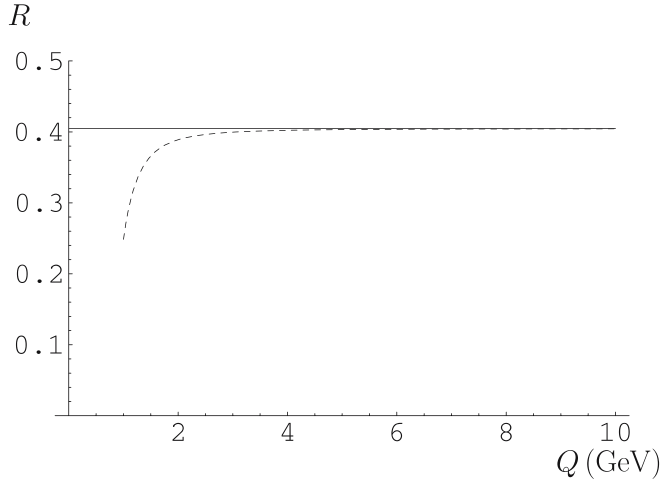

Fig. 2. The ratio $R(Q^2)$ of the squared amplitudes for H and π^0 production in $\gamma^*\gamma$ collisions at leading twist and zero-th order in α_s (*solid line*) and including twist three contributions in the numerator (*dashed line*)

where we average (sum) over the initial (final) lepton polarizations and use the equivalent photon approximation (EPA) for the leptonic part connected to the almost real photon,

$$\frac{d\sigma_{ee\to ee\,\pi\eta}}{dQ^2\,dW^2\,d\cos\theta\,d\varphi\,dx_2} = \frac{\alpha}{\pi}\frac{1}{x_2}\left(\frac{1+(1-x_2)^2}{2}\ln\left[\frac{Q'^2_{\max}(x_2)}{Q'^2_{\min}(x_2)}\right] - (1-x_2)\right) \times \frac{d\sigma_{e\gamma\to e\,\pi\eta}}{dQ^2\,dW^2\,d\cos\theta\,d\varphi}\,, \quad (25)$$

where $W = m_{\pi\eta}$ and $Q'^2_{\min}$ and $Q'^2_{\max}$ are the minimal and maximal virtuality of the photon q', respectively. For a given ee collider energy, the variables $x_2 = q'p/l_1p = s_{e\gamma}/s_{ee}$ and $y = qq'/k_1q'$ are not independent at fixed Q^2 and W^2, since $yx_2 = (Q^2 + W^2)/s_{ee}$. φ is defined as being the angle between leptonic and hadronic planes. The lower kinematical limit $Q'^2_{\min} = x_2^2\, m_e^2/(1-x_2)$ is determined by the electron mass m_e, whereas $Q'^2_{\max}$ depends on experimental cuts. Without a precise knowledge of these cuts, we will present results for the $e\gamma \to e\,\pi\eta$ process. Its cross section reads

$$\frac{d\sigma_{e\gamma\to e\pi\eta}}{dQ^2\,dW^2\,d\cos\theta\,d\varphi} = \frac{\alpha^3}{16\pi}\frac{\beta}{s_{e\gamma}^2}\frac{1}{Q^2}\cdot\frac{1}{2}\left(Q_u^2 - Q_d^2\right)^2 \quad (26)$$
$$\left(\frac{1+(1-y)^2}{4y^2}|A_1^{(\pi\eta)}|^2 + \frac{2\bar{y}\beta^2W^2}{Q^2y^2}\sin^2\theta|A_2^{(\pi\eta)}|^2 + \frac{\sqrt{1-y}\beta W(2-y)}{Qy^2}\cos\varphi\sin\theta\,\mathrm{Re}\left(A_1^{(\pi\eta)}A_2^{(\pi\eta)*}\right)\right).$$

To analyze the cross section, let us first define the cross section integrated over angles θ and φ, $d\sigma(Q^2, W^2)/dQ^2\,dW^2$

$$\frac{\mathrm{d}\sigma(Q^2,W^2)}{\mathrm{d}Q^2\,\mathrm{d}W^2} = \int \mathrm{d}\cos\theta\,\mathrm{d}\varphi \frac{\mathrm{d}\sigma_{\mathrm{e}\gamma\to\mathrm{e}\pi\eta}}{\mathrm{d}Q^2\,\mathrm{d}W^2\,\mathrm{d}\cos\theta\,\mathrm{d}\varphi} = \frac{25\alpha_{\mathrm{em}}^3\beta}{72 s_{\mathrm{e}\gamma}^2 Q^2}\cdot\frac{1+(1-y)^2}{y^2} \times\left[K^2+\frac{1}{3}a^2(W)b^2(W)\right], \quad (27)$$

where we restrict ourselves to the twist two contribution and

$$a(W) = \frac{5}{3} g_{H\pi\eta} f_H \beta M_H, \qquad b(W) = \frac{1}{\sqrt{\left(W^2-M_H^2\right)^2+\Gamma_H^2 M_H^2}}. \quad (28)$$

In (28) the coupling constant $g_{H\pi\eta}$ is related to the branching ratio $\mathrm{BR} = \Gamma_{H\to\pi\eta}/\Gamma_H$ of the hybrid meson by

$$g_{H\pi\eta}^2 = \frac{48\pi}{\beta^3 M_H}\Gamma_H \mathrm{BR}. \quad (29)$$

Since nothing is known about the value of the $H\to\pi\eta$ BR, in our numerical analysis we choose BR = 20% as a reasonable estimate. Let us also note that simultaneous rescaling of the background magnitude K and the BR by a coefficient c amounts to a rescaling of the cross sections by c^2, leaving the angular distribution $W(\theta,\varphi)$ (30) unchanged.

This cross section does not depend on the phase α appearing in our model of the $\pi\eta$–GDA, (14). The plots in Fig. 3 and Fig. 4 show its W-dependence for different magnitudes K of the assumed background, for $Q = 3$ GeV and $Q = 5$ GeV, respectively. We observe that the presence of the hybrid peak around $W = 1.4$ GeV is hardly affected when changing the magnitude K of the background. The magnitude of this signal is comparable with what has been achieved by the L3 experiment at LEP [5]. As expected, the comparison of Figs. 3 and 4 shows that the magnitude of the signal decreases when Q increases. However, this decrease is not so dramatic due to the scaling behaviour of the amplitude from twist two dominance. We thus expect that the Q-dependence may be experimentally studied up to a few GeV.

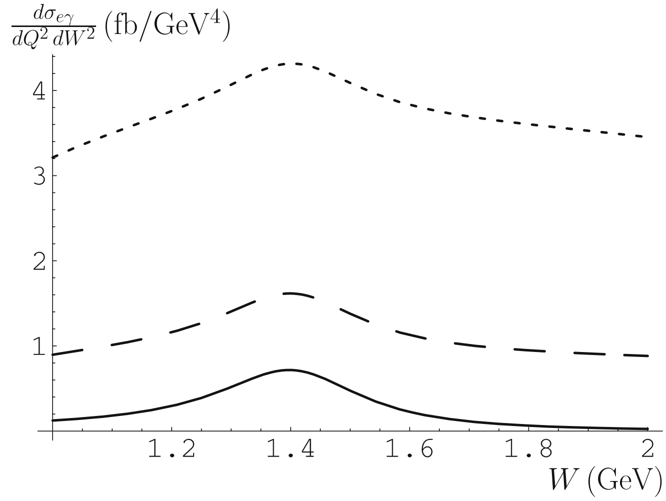

Fig. 4. The differential cross-section for $\pi\eta$ pair production as a function of W for $Q = 5$ GeV, $y = 0.3$, for different background magnitudes $K = 0$ (*solid curve*), 0.5 (*dashed curve*) and 1 (*dotted curve*)

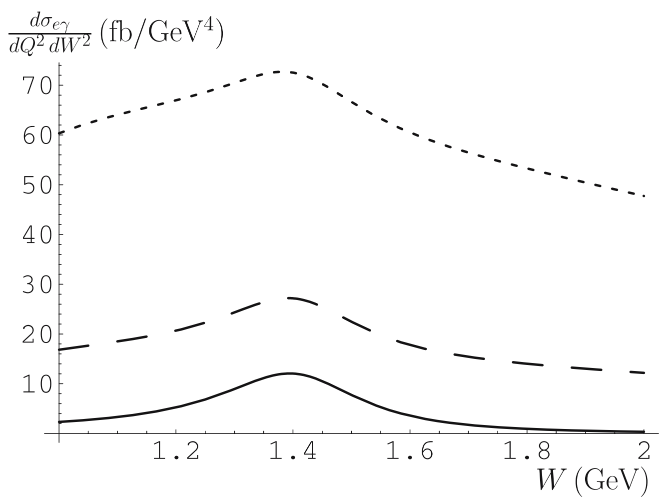

Fig. 3. The differential cross-section for $\pi\eta$ pair production as a function of W for $Q = 3$ GeV, $y = 0.3$, for different background magnitudes $K = 0$ (*solid curve*), 0.5 (*dashed curve*) and 1 (*dotted curve*)

A more detailed test of the nature of an eventual signal may be accessed by a study of the angular distribution of the $\pi\eta$ final state. Using (26) and (27) supplemented by (17), (20) and (21), we obtain

$$W(\theta,\phi) = \left(\frac{\mathrm{d}\sigma(Q^2,W^2)}{\mathrm{d}Q^2\,\mathrm{d}W^2}\right)^{-1}\frac{\mathrm{d}\sigma_{\mathrm{e}\gamma\to\mathrm{e}\pi\eta}}{\mathrm{d}Q^2\;\mathrm{d}W^2\;\mathrm{d}\cos\theta\;d\varphi} = \frac{1}{4\pi}\left[A+B\cos\theta+C\cos^2\theta + D\sin 2\theta\cos\phi + E\sin\theta\cos\phi\right], \quad (30)$$

with the functions $A(W)$, $B(W)$, $C(W)$, $D(W,Q)$ and $E(W,Q)$:

$$A = \frac{K^2}{K^2+\frac{1}{3}a^2b^2}, \quad B = \frac{2Kab\cos(\gamma-\alpha)}{K^2+\frac{1}{3}a^2b^2}, \quad C = \frac{a^2b^2}{K^2+\frac{1}{3}a^2b^2},$$
$$D = \frac{W}{Q}\frac{(2-y)\sqrt{1-y}}{3[1+(1-y)^2]}C, \quad E = \frac{W}{Q}\frac{(2-y)\sqrt{1-y}}{3[1+(1-y)^2]}B, \quad (31)$$

and where γ is the phase of the Breit–Wigner form, i.e.

$$\cos\gamma = \left(M_H^2-W^2\right)b(W), \qquad \sin\gamma = b(W)\Gamma_H M_H. \quad (32)$$

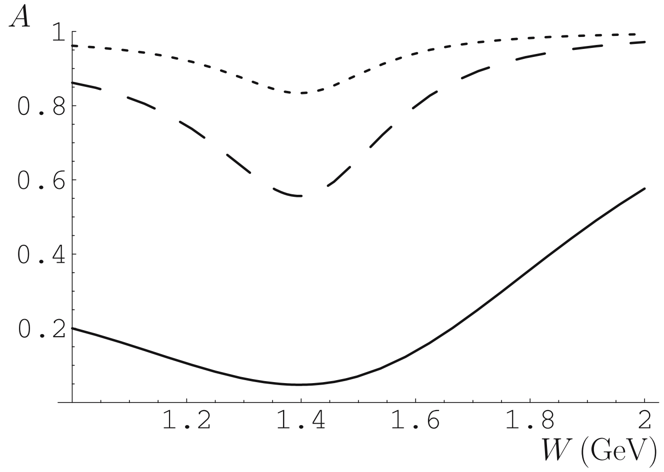

Fig. 5. The A component of the angular distribution function as a function of the $\pi\eta$ mass W for $Q = 3$ GeV, $y = 0.3$, for different background magnitudes $K = 0.1$ (*solid curve*), 0.5 (*dashed curve*) and 1 (*dotted curve*)

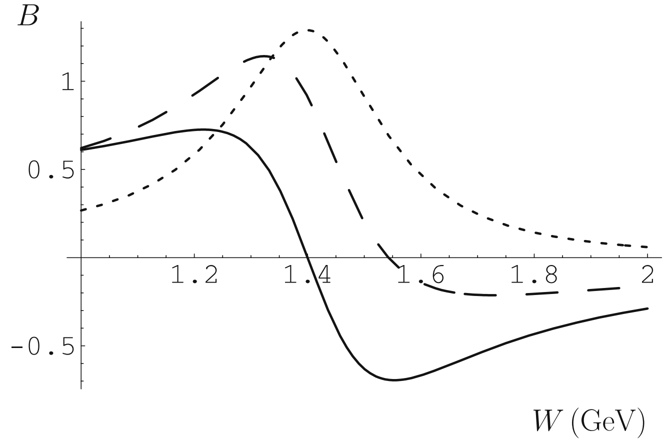

Fig. 7. The B component of the angular distribution function as a function of the $\pi\eta$ mass W for $Q = 3$ GeV, $y = 0.3$ and $K = 1$, for different background phases $\alpha = 0$ (*solid curve*), $\pi/4$ (*dashed curve*) and $\pi/2$ (*dotted curve*)

In (31) we restrict ourselves to the twist two contributions for the coefficients $A(W)$, $B(W)$, $C(W)$, which is why A, B, C are Q-independent. Due to the normalization of the angular distribution $W(\theta,\varphi)$, $\int \mathrm{d}\cos\theta\,\mathrm{d}\varphi W(\theta,\varphi) = 1$, the functions A and C are not independent but they satisfy the relation $A + C/3 = 1$. The function A is displayed in Fig. 5. Its shape is dictated by the inverse shape of the integrated cross-section shown in Fig. 3, as seen from (31) and (27). This is intimately related to our main assumption that the background is described by a $J = 0$ contribution.

The B coefficient is displayed in Figs. 6 and 7, for a background magnitude $K = 1$ and $y = 0.3$. It measures the interference between the background and the hybrid signal. It is thus quite dependent on the value of the phase of the background, but its W-dependence always reveals a dramatic change around the mass of the hybrid meson. One may use it to determine this mass more precisely.

Since D and E vanish at the twist two level, this part of the angular distribution of the final mesons is sensitive to the strength of the twist three amplitude. We calculate them in the Wandzura–Wilczek approximation described in Sect. 2. The coefficient D is independent of the phase of the background. In Fig. 8 we show its behaviour when the magnitude of the background varies, for $Q = 3$ GeV and $y = 0.3$. In Fig. 9 we show its W-dependence for $K = 1$ and $y = 0.3$ and for three different values of Q. Because of its proportionality with C, and thus its relation to A, it is strongly peaked around the hybrid mass. It will only be measurable at fairly small values of Q. $E(W,Q)$ depends on the background phase α, as does B, and on the magnitude K. In Figs. 10 and 11 we show its W-dependence for

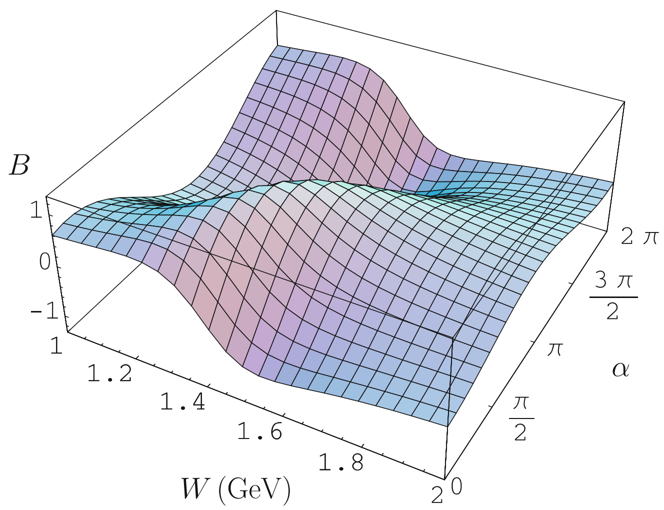

Fig. 6. The B component of the angular distribution function as a function of the $\pi\eta$ mass W and of the background phase α for the background magnitude $K = 1$, with $Q = 3$ GeV and $y = 0.3$

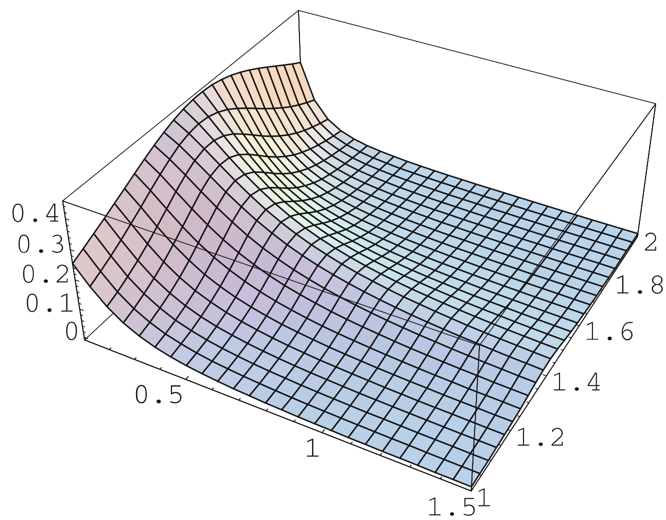

Fig. 8. The D component of the angular distribution function as a function of the $\pi\eta$ mass W and of the background magnitude K for $Q = 3$ GeV and $y = 0.3$

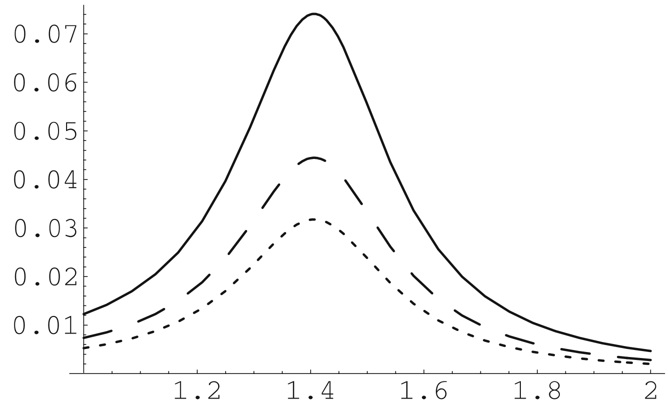

Fig. 9. The D component of the angular distribution function as a function of the $\pi\eta$ mass W for $y = 0.3$ and $K = 1$, for different values of $Q = 3$ GeV (*solid curve*), 5 GeV (*dashed curve*) and 7 GeV (*dotted curve*)

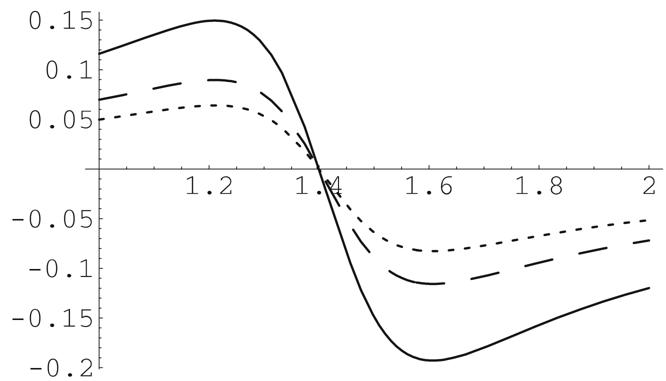

Fig. 10. The E component of the angular distribution function as a function of the $\pi\eta$ mass W for $y = 0.3$ and $K = 0.5$, for different values of $Q = 3$ GeV (*solid curve*), 5 GeV (*dashed curve*) and 7 GeV (*dotted curve*)

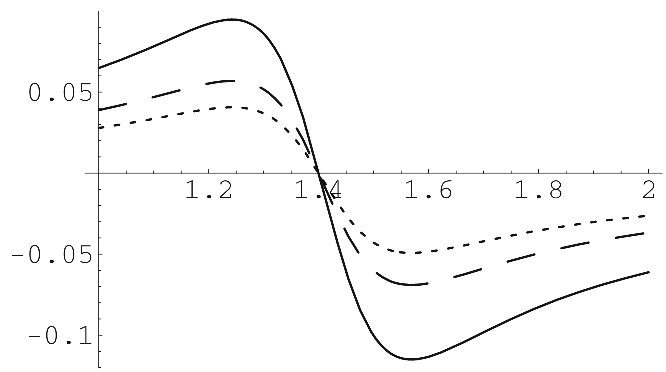

Fig. 11. The E component of the angular distribution function as a function of the $\pi\eta$ mass W for $y = 0.3$ and $K = 1$, for different values of $Q = 3$ GeV (*solid curve*), 5 GeV (*dashed curve*) and 7 GeV (*dotted curve*)

$\alpha = 0$ and $K = 0.5$ and $K = 1$, respectively. This behaviour is similar to the one of B displayed in Fig. 7, but its magnitude is much smaller when Q is greater than 5 GeV.

5 Single spin asymmetry

We consider now the exclusive process where a longitudinally polarized lepton (with helicity h) scatters on an unpolarized photon to produce the lepton and the hybrid meson detected through its decay into a $\pi\eta$ pair. Such a process allows us to define an asymmetry that is zero at the leading twist level but receives contributions from the interference of twist two and twist three amplitudes. This asymmetry is related to the azimuthal angular dependence of the polarized cross section and it is defined as

$$\mathcal{A}_1(s_{e\gamma}, Q^2, W^2; \varphi) = \frac{\int \mathrm{d}\cos\theta_{cm}(\mathrm{d}\sigma^{(\rightarrow)} - \mathrm{d}\sigma^{(\leftarrow)})}{\int \mathrm{d}\cos\theta_{cm}(\mathrm{d}\sigma^{(\rightarrow)} + \mathrm{d}\sigma^{(\leftarrow)})}\,, \tag{33}$$

where by $\mathrm{d}\sigma^{(\rightarrow)}$ we denote the differential cross section $\mathrm{d}\sigma^{(h=1)}_{e\gamma\to e\pi\eta}/\,\mathrm{d}W^2\,\mathrm{d}Q^2\,\mathrm{d}\cos\theta_{cm}\,\mathrm{d}\varphi$.

Note that the denominator (we restrict ourselves to the dominant twist two component) can be expressed through the unpolarized differential cross section defined in (27)

$$\int_0^{2\pi} \mathrm{d}\,\varphi \int_{-1}^{1} \mathrm{d}\cos\theta_{cms}\left(\mathrm{d}\sigma^{(\rightarrow)} + \mathrm{d}\sigma^{(\leftarrow)}\right) = 2\frac{\mathrm{d}\sigma_{e\gamma}(Q^2, W^2)}{\mathrm{d}Q^2\,\mathrm{d}W^2}\,. \tag{34}$$

The asymmetry (33) reads:

$$\mathcal{A}_1(s_{e\gamma}, Q^2, W^2; \varphi) = \frac{\int_0^{\pi} \mathrm{d}\theta_{cm}\,\sin\theta_{cm}\,\mathcal{N}(\theta_{cm}, Q, W, \varphi)}{2\int_0^{\pi} \mathrm{d}\theta_{cm}\sin\theta_{cm}\,\mathcal{D}(\theta_{cm})}\,, \tag{35}$$

where

$$\mathcal{N}(\theta_{cm}, Q, W, \varphi) = \frac{4}{Q^6}\varepsilon_{k_1 q' k_2 \Delta_T^{(\pi\eta)}}\mathrm{Im}\left(\mathbf{A}_1^{(\pi\eta)}\mathbf{A}_2^{*(\pi\eta)}\right)\,,$$
$$\mathcal{D}(\theta_{cm}) = \frac{1}{Q^2}\frac{1+(1-y)^2}{4y^2}|\mathbf{A}_1^{(\pi\eta)}|^2\,. \tag{36}$$

The crucial dynamical quantity probed by this asymmetry is thus $\mathrm{Im}(\mathbf{A}_1^{(\pi\eta)}\mathbf{A}_2^{*\,(\pi\eta)})$. It depends on the phase structure of the amplitudes, and may be written as

$$\mathrm{Im}\left(\mathbf{A}_1^{(\pi\eta)}\mathbf{A}_2^{*(\pi\eta)}\right) = \frac{50}{3}\frac{a(W)b(W)K}{\beta}\sin(\alpha-\gamma)\,, \tag{37}$$

where $a(W)$, $b(W)$ and γ are defined by (28) and (32). This quantity depends much on the unknown background phase α. In Fig. 12 we present our result with the choice $\alpha = 0$. The resulting asymmetry is sizeable and should be measurable.

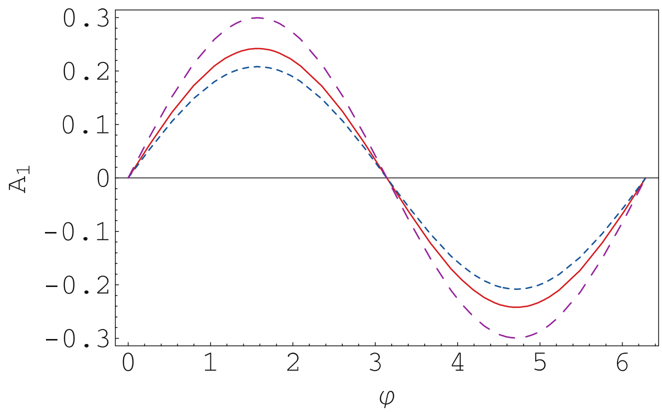

Fig. 12. The single spin asymmetry $\mathcal{A}_1$ as function of $\varphi = (0,\ 2\pi)$. Values of parameters: $W = 1.4\,\mathrm{GeV}, Q^2 = 5.0\,\mathrm{GeV}^2$, $s_{e\gamma} = 10\,\mathrm{GeV}^2$, $\alpha = 0$. The solid line corresponds to $K = 0.8$, the *short-dashed line* to $K = 1.0$, the *long-dashed line* to $K = 0.5$

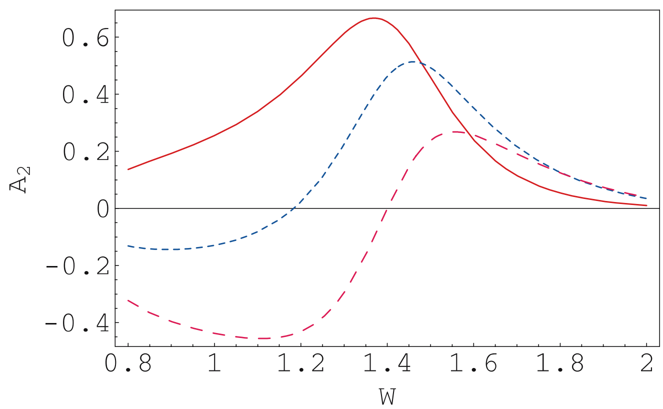

Fig. 13. The integrated single spin asymmetry $\mathcal{A}_2$ as a function of the $\pi\eta$ invariant mass. Values of parameters: $Q^2 = 5.0\,\mathrm{GeV}^2$, $s_{e\gamma} = 10\,\mathrm{GeV}^2$, $K = 1.0$, the solid line corresponds to $\alpha = 0$, the *short-dashed line* to $\alpha = \pi/4$, the *long-dashed line* to $\alpha = \pi/2$

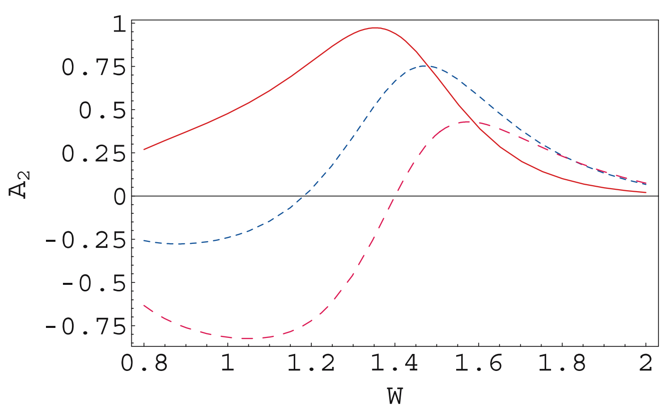

Fig. 14. The integrated single spin asymmetry $\mathcal{A}_2$ as a function of the $\pi\eta$ invariant mass. Values of parameters: $Q^2 = 5.0\,\mathrm{GeV}^2$, $s_{e\gamma} = 10\,\mathrm{GeV}^2$, $K = 0.5$, the solid line corresponds to $\alpha = 0$, the *short-dashed line* to $\alpha = \pi/4$, the *long-dashed line* to $\alpha = \pi/2$.

In order to obtain more statistics, one may define an integrated asymmetry:

$$\mathcal{A}_2(s_{e\gamma}, Q^2, W^2) = \frac{2\pi \int \mathrm{d}(\cos\theta_{cm}) \int\limits_0^{2\pi} \mathrm{d}\varphi\, \sin\varphi (\mathrm{d}\sigma^{(\rightarrow)} - \mathrm{d}\sigma^{(\leftarrow)})}{\int \mathrm{d}(\cos\theta_{cm}) \int\limits_0^{2\pi} \mathrm{d}\varphi\, (\mathrm{d}\sigma^{(\rightarrow)} + \mathrm{d}\sigma^{(\leftarrow)})}, \tag{38}$$

which may be used as a probe of the $\pi\eta$ invariant mass dependence. In Figs. 13 and 14 we show the resulting W-dependence for two choices of the background magnitude K.

6 Conclusion

This theoretical study shows that if a hybrid meson exists with $J^{PC} = 1^{-+}$ around 1.4 GeV and with a sizeable branching ratio to $\pi - \eta$,, much can be learned about it from the experimental observation of $\gamma^*\gamma$ reactions and the precise study of the $\pi - \eta$ final state. The magnitude of the cross section that we obtain in our model of the $\pi - \eta$ generalized distribution amplitude indicates that present detectors at current e^+e^- colliders are able to obtain good statistics on these reactions, provided that the tagging procedure is efficient.

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